Lomené výrazy 04

1. Zjednodušte a určete podmínky:
$$\left(1+\frac{1}{a^2-1}\right):\left[\frac{1}{a+1}-(1-a)\right]=$$

$$\left(1+\frac{1}{a^2-1}\right):\left[\frac{1}{a+1}-(1-a)\right]=\frac{a^2-1+1}{a^2-1}:\frac{1-(1-a)(1+a)}{a+1}=\frac{a^2}{a^2-1}:\frac{1-\left(1-a^2\right)}{a+1}=$$

$$=\frac{a^2}{a^2-1}:\frac{1-1+a^2}{a+1}=\frac{a^2}{(a-1)(a+1)}\cdot\frac{a+1}{a^2}=\frac{1}{a-1} \qquad \text{Podmínky: } a\neq\pm1; a\neq0$$

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2. Zjednodušte a určete podmínky:
$$\left(\frac{x+1}{x+2}-\frac{x-1}{x-2}\right):\frac{2x}{4-x^2}=$$

$$\left(\frac{x+1}{x+2}-\frac{x-1}{x-2}\right):\frac{2x}{4-x^2}=\frac{(x+1)(x-2)-(x-1)(x+2)}{(x+2)(x-2)}:\frac{2x}{(2-x)(2+x)}=$$

$$=\frac{x^2-2x+x-2-\left(x^2+2x-x-2\right)}{(x+2)(x-2)}\cdot\frac{(2-x)(2+x)}{2x}=\frac{x^2-x-2-x^2-2x+x+2}{(x+2)(x-2)}\cdot\frac{(2-x)(2+x)}{2x}=$$

$$=\frac{-2x}{(x+2)(x-2)}\cdot\frac{-(x-2)(2+x)}{2x}=+1 \qquad \text{Podmínky: } x\neq\pm2; x\neq0$$

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3. Zjednodušte a určete podmínky:
$$\frac{\dfrac{a}{a-2}-\dfrac{a}{a+2}}{\dfrac{4a}{a^2(2a-5)-4(2a-5)}}=$$

$$\frac{\dfrac{a}{a-2}-\dfrac{a}{a+2}}{\dfrac{4a}{a^2(2a-5)-4(2a-5)}}=\left(\frac{a}{a-2}-\frac{a}{a+2}\right):\left(\frac{4a}{a^2(2a-5)-4(2a-5)}\right)=$$

$$=\frac{a(a+2)-a(a-2)}{(a-2)(a+2)}:\frac{4a}{(2a-5)(a^2-4)}=\frac{a^2+2a-a^2+2a}{(a-2)(a+2)}:\frac{4a}{(2a-5)(a-2)(a+2)}=$$

$$=\frac{4a}{(a-2)(a+2)}\cdot\frac{(2a-5)(a-2)(a+2)}{4a}=2a-5 \qquad \text{Podmínky: } a\neq\pm2; a\neq\frac{5}{2}; a\neq0$$

4. Zjednodušte a určete podmínky:

$$\frac{2x-\dfrac{1+4x}{3}}{2x-\dfrac{1+8x}{5}}=$$

$$\frac{2x-\dfrac{1+4x}{3}}{2x-\dfrac{1+8x}{5}}=\frac{\dfrac{6x-1-4x}{3}}{\dfrac{10x-1-8x}{5}}=\frac{\dfrac{2x-1}{3}}{\dfrac{2x-1}{5}}=\frac{2x-1}{3}\cdot\frac{5}{2x-1}=\frac{5}{3}\qquad\text{Podmínky: }x\neq\frac{1}{2}$$

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5. Zjednodušte a určete podmínky:

$$\frac{\dfrac{5-a}{a^2}}{\dfrac{3}{a}}:\left[\frac{4}{3a}+\frac{1}{3\cdot(a-2b)}\cdot\frac{a(2b-a)-(2b-a)}{a}\right]=$$

$$\frac{\dfrac{5-a}{a^2}}{\dfrac{3}{a}}:\left[\frac{4}{3a}+\frac{1}{3\cdot(a-2b)}\cdot\frac{a(2b-a)-(2b-a)}{a}\right]=$$

$$=\frac{(5-a)\cdot a}{3a^2}:\left[\frac{4}{3a}+\frac{1}{3\cdot(a-2b)}\cdot\frac{(2b-a)^{-1}\cdot(a-1)}{a}\right]=$$

$$=\frac{5-a}{3a}:\left[\frac{4}{3a}-\frac{a-1}{3a}\right]=\frac{5-a}{3a}:\frac{4-a+1}{3a}=\frac{5-a}{3a}:\frac{5-a}{3a}=\frac{5-a}{3a}\cdot\frac{3a}{5-a}=1$$

Podmínky: $a\neq 0; a\neq 2b; a\neq 5$

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6. Zjednodušte a určete podmínky:

$$\frac{4x-6x^2}{\left(2x-\dfrac{1}{2}\right)^2-\left(x-\dfrac{3}{2}\right)^2}=$$

$$\frac{4x-6x^2}{\left(2x-\dfrac{1}{2}\right)^2-\left(x-\dfrac{3}{2}\right)^2}=\frac{2x(2-3x)}{\left[\left(2x-\dfrac{1}{2}\right)-\left(x-\dfrac{3}{2}\right)\right]\cdot\left[\left(2x-\dfrac{1}{2}\right)+\left(x-\dfrac{3}{2}\right)\right]}=$$

$$=\frac{2x(2-3x)}{\left[2x-\dfrac{1}{2}-x+\dfrac{3}{2}\right]\cdot\left[2x-\dfrac{1}{2}+x-\dfrac{3}{2}\right]}=\frac{2x(2-3x)^{(-1)}}{[x+1]\cdot[3x-2]}=\frac{-2x}{x+1}\qquad\text{Podmínky: }x\neq -1; x\neq\frac{2}{3}$$

7. Zjednodušte a určete podmínky:
$$\left(\frac{1-3m}{m^2-1}+\frac{m}{m-1}\right):\frac{2(m+2)}{m^3+2m^2-m-2}=$$

$$\left(\frac{1-3m}{m^2-1}+\frac{m}{m-1}\right):\frac{2(m+2)}{m^3+2m^2-m-2}=\left(\frac{1-3m}{(m-1)(m+1)}+\frac{m}{m-1}\right):\frac{2(m+2)}{m^2(m+2)-(m+2)}=$$

$$=\frac{1-3m+m(m+1)}{(m-1)(m+1)}:\frac{2(m+2)}{(m+2)(m^2-1)}=\frac{1-3m+m^2+m}{(m-1)(m+1)}:\frac{2(m+2)}{(m+2)(m-1)(m+1)}=$$

$$=\frac{m^2-2m+1}{(m-1)(m+1)}\cdot\frac{(m-1)(m+1)}{2}=\frac{(m-1)^2}{(m-1)(m+1)}\cdot\frac{(m-1)(m+1)}{2}=\frac{(m-1)^2}{2}$$

Podmínky: $m\neq\pm1, m\neq-2$

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8. Zjednodušte a určete podmínky:
$$\frac{\dfrac{a^2+b^2+2ab}{b}}{\dfrac{a+b}{ab}}+\frac{\dfrac{2ab-a^2-b^2}{a}}{\dfrac{a-b}{ab}}=$$

$$\frac{\dfrac{a^2+b^2+2ab}{b}}{\dfrac{a+b}{ab}}+\frac{\dfrac{2ab-a^2-b^2}{a}}{\dfrac{a-b}{ab}}=\frac{a^2+2ab+b^2}{b}\cdot\frac{ab}{a+b}+\frac{-(a^2-2ab+b^2)}{a}\cdot\frac{ab}{a-b}=$$

$$=\frac{(a+b)^2}{b}\cdot\frac{ab}{a+b}-\frac{(a-b)^2}{a}\cdot\frac{ab}{a-b}=a^2+ab-ab+b^2=a^2+b^2 \qquad \text{Podmínky: } a\neq0; b\neq0; a\neq\pm b$$